Effects of dissipation on the time evolution of coherence and entanglement of a two-quantum dot system in a microcavity

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Abstract. The effects of dissipation on a system composed by two quantum-dot qubits interacting with a single mode of light in a microcavity are studied by computing the time evolution of mixedness and entanglement of the qubits and the second order correlation function of the field.

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Introduction. Excitons in semiconductor quantum dots embedded in microcavities exhibit all the good characteristics of quantum bits. Although there are other systems with the same characteristics, such as Rydberg atoms in QED cavities, quantum dots have a size advantage and can be easily manipulated. However, in semiconductor microcavities quality factors are much lower (of the order of $10^3 - 10^4$) and, therefore, a complete study of how dissipation affects the quantum properties of the qubits is necessary. We consider a standard situation of quasi-two dimensional quantum dots, modelled as two-level systems (TLS) [1], embedded in a λ pillar microcavity. The non-interacting dots are resonantly coupled to a single cavity field mode [2]. The model is simple but leads to interesting phenomena.

Model. The interaction between a single mode of the electromagnetic field and N excitons is described by the Tavis-Cummings hamiltonian: $H = \sum_{i=1}^{N}\left(\frac{1}{2}\hbar\omega_i\sigma_{z_i}\right) + \hbar\omega_0 a^\dagger a + \sum_{i=1}^{N}\left\{\hbar g_i\left(a^\dagger\sigma_i^- + a\sigma_i^+\right)\right\}$, where the first two terms are the free exciton and photon energies respectively, $\sigma_i^+ = |+i\rangle\langle -i|$, $\sigma_i^- = |-i\rangle\langle +i|$ are the excitonic ladder operators, and a ($a^\dagger$) are the anihilation (creation) field operators. In order to include the effects of the environment we must write the master equation for the system's density operator in the Born-Markov approximation. The general form of this equation is: $\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \mathscr{L}(\rho)$, where $\mathscr{L}$ describes the non-conservative part of the evolution.

In general, there are many non-conservative processes that can be considered [1], like exciton relaxations, coherent and incoherent pumping or emission, etc. In the present paper we take account only of coherent emission due to the incomplete reflectance of the mirrors, and spontaneous emission from excitons will not be considered here because it can be inhibited by engineering the cavity, via Purcell effect [3]. Our dissipative Liouvillian is therefore $\mathscr{L}(\rho) = \kappa\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$. From the time evolution of the density operator, we can compute the populations, coherences and expectation values of any observable.

Characterizing the dynamics. We will use two observables in order to study the effects of dissipation on the dynamics of the two qdots (subsystem A). The first quantity, which characterizes the mixedness of the excitons, is the linear entropy [4], $M_A = 1 - Tr(\rho_A^2)$, where ρ_A is the reduced density matrix of the excitons. The second quantity is the entanglement between the two qubits, which can be measured by the Wootters $C(\rho_A)$ concurrence function [5]: $C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where λ_i are the eigenvalues in decreasing order of the matrix $R = \sqrt{\sqrt{\rho}(\sigma_y\otimes\sigma_y)\rho^\star(\sigma_y\otimes\sigma_y)\sqrt{\rho}}$. On the other hand, the correlation functions measure the coherence properties of the electromagnetic field [6]. They characterize photon statistics (value of $g^{(2)}$) and the spectrum of emitted light (the Fourier transform of $g^{(1)}$). The second order correlation function for a single-mode quantized field reduces to [7]: $g^{(2)}(t) = \langle\hat{N}(\hat{N}-1)\rangle / \langle\hat{N}\rangle^2$.

Results. Several types of initial conditions can be chosen: entangled qdots, mixed qdots, globally entangled states, globally mixed states. In terms of the usual computational basis for the two qubits ($|1\rangle = |{+}{+}\rangle$, $|2\rangle = |{+}{-}\rangle$, $|3\rangle = |{-}{+}\rangle$ and $|4\rangle = |{-}{-}\rangle$), the following particular initial conditions were considered. Maximally entangled state of the qdots ($E+$) –Bell state–:

$$\rho_{E+} = 1/2(|2\rangle\langle 2| + |2\rangle\langle 3| + |3\rangle\langle 2| + |3\rangle\langle 3|)\otimes|2\rangle\langle 2|.$$

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and Maximally mixed state of the qdots ($M+$) –Thermal equilibrium state–:

$$\rho_{M+} = 1/4(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| + |4\rangle\langle 4|) \otimes |2\rangle\langle 2|.$$

In general we can define three matter-light regimes: Strong, Medium and Weak coupling. Since we are interested in the evolution of coherences and exciton-photon correlations, and taking into account that dissipation dominates the dynamics in the weak coupling regime, strongly suppressing the coherences, we consider only the strong coupling regime $g/\kappa \sim 10$. In what follows the frequencies of the two qubits are set to ω and the coupling constants to $g = 1$. For all calculations a resonance condition is imposed, i.e., $\omega = \omega_0 = 10^3$.

Figure 1 shows the evolution of C, M, $g^{(2)}$ for the initial conditions E+ and M+. The first one represents the behaviour of System-entangled states while M+ represents the behaviour of System-separable states. For System separable states (M+) the concurrence is zero at all times. The linear entropy and the second order correlation function $g^{(2)}$ oscillates. For these initial states $g^{(2)} > 1$ at some point, therefore light emmited from this system may have quantum or classical behaviour [7] at different times depending on the initial conditions and that there is no relation between the three quantities.

For system entangled states (E+) the three quantities C, M and $g^{(2)}$ and the correlations between them change at a transition time t_C around 7 ps. Concurrence presents a collapse and revival behaviour in all the range, but after t_C the oscillations lost defined frequency and are strongly suppressed. Linear entropy oscillates before time t_C and decreases monotonically from that point. The second order correlation function oscillates uniformly with a constant mean value, and then starts to decrease with double peaks. Note that concurrence, linear entropy and the second order correlation function are very correlated before t_c: concurrence and linear entropy oscillate in counter-phase and for initial times the maximum purity is reached at half the time between two revivals of concurrence. The second order correlation function oscillates in phase with the linear entropy and has a greater amplitude.

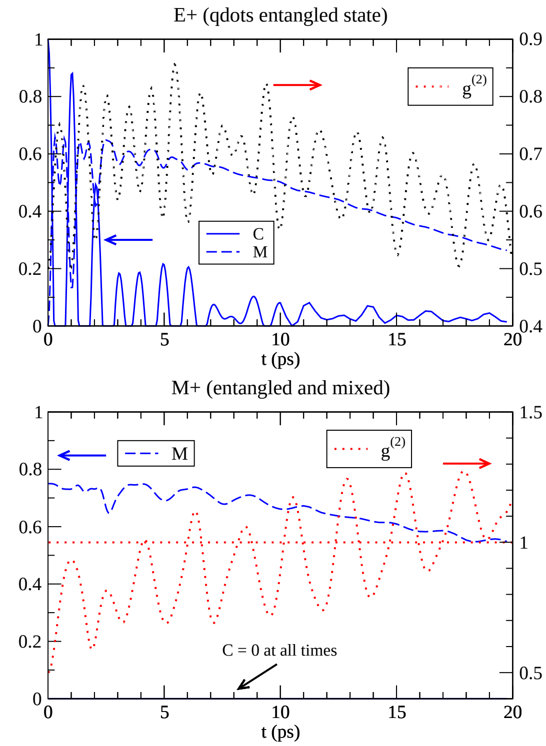

FIGURE 1. Concurrence C, Linear entropy M and Second order correlation function $g^{(2)}$ for E+ and M+ in the strong coupling regime.

Summary. Throughout this work we have characterized linear entropy, concurrence and second order correlation function of a two quantum dot system in a microcavity in terms of the regime, defined by the coupling of the system with the field g and the dissipation rate κ, and the initial conditions. We found that a transition regime, for system entangled states, may be identified in the strong coupling with these three functions $(C, M, g^{(2)})$ which are highly correlated at the beginning and lost their well defined phase relations in the dynamics. This characterization was made, on purpose, in terms of observables that are accessible by experiment. We cannot measure linear entropy and concurrence of the quantum dots; only emitted light and therefore the second order correlation function can be obtained.

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