

## Decoherence of mesoscopic states of cavity fields

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We show that two-atom correlation measurements of the type involved in a recent experimental study of the evolution of a mesoscopic superposition state prepared in a definite mode of a high- $Q$  cavity can be used to determine the eigenvalues of the reduced density matrix of the field, provided the assumed dynamical conditions are actually fulfilled to experimental accuracy. These conditions involve (i) a purely dispersive coupling of the field to the Rydberg atoms used to manipulate and to monitor the cavity field, and (ii) the effective absence of correlations in the ground state of the system consisting of the cavity coupled to the “reservoir” which accounts for the decoherence and damping processes. A microscopic calculation at zero temperature is performed and compared to master equation results. [S1050-2947(98)08309-7]

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The conception of an experimental setup to produce a superposition of mesoscopic quantum states and to observe its subsequent decoherence [1] in microwave cavities has been preceded by a series of meticulous experimental and theoretical studies [2–5]. Several schemes have been proposed, based on two-atom correlation measurements involving circular Rydberg atoms, and the actual implementation of one such scheme has been recently reported [6]. It can be broadly pictured as involving two successive stages, which consist, respectively, in the preparation of the desired superposition state and in the subsequent monitoring of its time evolution under the influence of the unavoidable loss processes present in the cavity. As the last step in the preparation stage, what is now often referred to as the “reduction postulate” [7] is used instrumentally in an essential way to select the desired state of the field (see below). The subsequent stage, however, involves just strictly causal quantum evolution laws, and is therefore, in principle, amenable to a complete theoretical description, including the coherence loss of the prepared superposition state [2,5]. In particular, as it is appropriate for bona fide physical properties and processes, the coherence loss should be characterized and described in a basis independent fashion. We show in continuation that this can be done in a particularly simple way in the present case, given the specific properties of the prepared superposition state, and no more than ordinary assumptions concerning the external couplings underlying the mechanism of coherence loss. More specifically, we show that, under the experimental conditions stated in [6], two-atom correlations of the type involved in the measurements reported there can be used in order to determine completely the eigenvalues of reduced density matrices which describe the possible quantum states of the cavity field following the preparation stage. The time dependence of the observed two-atom correlations subsequently to the preparation of the superposition state constitutes therefore a measurement of intrinsic features of the evolution of the field which bear directly on the coherence loss process. Moreover, to the extent that this evolution depends both on the nature of the initial state and on dynamical assumptions concerning the dissipative process, through

an analysis of the type proposed here such measurements can provide *experimental* checks on the actual outcome of the preparation stage and on the adequacy of the assumed dissipative dynamics, at no higher cost than a less inclusive analysis of the two-atom correlation data.

The experiment reported in [6] involves a high- $Q$  cavity  $C$  located between two low- $Q$  cavities (Ramsey zones  $R_1$  and  $R_2$ ) fed with classical fields which steer the internal state of the crossing circular Rydberg atoms. The transition between two near atomic levels, usually denoted as  $|e\rangle$  and  $|g\rangle$ , is resonant with the fields in cavities  $R_1$  and  $R_2$ . Each atom is initially prepared in the state  $|e\rangle$ . After leaving  $R_1$  it is in a superposition of the states  $|e\rangle$  and  $|g\rangle$ . The fields in  $R_1$  and in  $R_2$  (and their relative phases) are chosen so that their action on the atoms is given in both cases by the unitary transformation  $U_R$ :

$$|e\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle), \quad |g\rangle \rightarrow \frac{1}{\sqrt{2}}(-|e\rangle + |g\rangle).$$

The high- $Q$  cavity  $C$  stores a coherent field  $|\alpha\rangle$ . The coupling between the atom and the field in this cavity is measured by the Rabi frequency  $\Omega$ . Two situations will be considered: (a) the frequency  $\omega$  of a relevant mode of  $C$  is nearly resonant with the transition frequency  $\omega_{ie}$  connecting  $|e\rangle$  to a third state  $|i\rangle$  (assumed for definiteness to lie above  $|e\rangle$  in energy) and is far off resonance with any transitions involving level  $|g\rangle$  [5]; and (b)  $\omega$  is nearly resonant with the transition frequency  $\omega_{eg}$  connecting  $|e\rangle$  to  $|g\rangle$  [2,6]. In both cases the detuning  $\delta$  is assumed large enough so that real transitions are effectively hindered and the atom-field interaction leads essentially to  $1/\delta$  dispersive frequency shifts. In this way, the atom-field coupling during a time  $t$  produces an atomic level dependent dephasing of the field and thus generates entangled states of the atom and field. This can be represented by the unitary operators

$$\begin{aligned} U_C^{(a)} &= e^{-i\phi a^\dagger a} |e\rangle\langle e| + |g\rangle\langle g| \quad [\text{case (a)}], \\ U_C^{(b)} &= e^{i\phi(a^\dagger a + 1)} |e\rangle\langle e| + e^{-i\phi a^\dagger a} |g\rangle\langle g| \quad [\text{case (b)}], \end{aligned} \quad (1)$$

with  $\phi(t) = \Omega^2 t / \delta$ . In these expressions the operators  $a^\dagger$ ,  $a$  create and annihilate photons in the relevant mode of  $C$ . The effect of these operators on a coherent state of this cavity amounts thus simply to a change of *phase* which depends on the state of the intervening two-level atom, so that the resulting state of the field can be expressed in terms of just two coherent states. It should be emphasized that this feature depends crucially on the validity of the purely dispersive limit mentioned above, which requires values of  $\Omega/\delta$  sufficiently small to ensure the validity of the implied low-order perturbative treatment [8].

This idealization of the involved dynamics allows one to describe the experiment in a symbolically compact form. When an atom is sent through the apparatus, the change in the state of the combined system consisting of atom plus field is described by  $U_R U_C U_R$ , so that for a factorized initial state  $\rho^F \otimes |e\rangle\langle e|$  (with  $\rho^F = |\alpha\rangle\langle\alpha|$ ) one has

$$U_R U_C U_R (\rho^F \otimes |e\rangle\langle e|) U_R^\dagger U_C^\dagger U_R^\dagger$$

for the combined system. The field state resulting from the final observation of the atom in state  $|e\rangle$  or  $|g\rangle$  is obtained by replacing the product of unitary operators, respectively, by the reduced operators

$$\mathcal{U}_g^e \equiv \langle e| U_R U_C U_R |e\rangle \quad (2)$$

(or their adjoints) with an appropriate trace normalization factor. This is what we referred to as the preparation stage of the experiment. The instrumental role attributed to the ‘reduction postulate’ in this stage appears explicitly in this last step. One gets, for the two cases considered,

$$\mathcal{U}_g^{(a)} = \frac{1}{2} [e^{-i\phi a^\dagger a} \mp 1]$$

and

$$\mathcal{U}_g^{(b)} = \frac{1}{2} [e^{i\phi(a^\dagger a + 1)} \mp e^{-i\phi a^\dagger a}]. \quad (3)$$

The indices  $e, g$  and the double signs refer, respectively, to the final observation of the atom in states  $|e\rangle$  and  $|g\rangle$ .

We now turn to the second stage of the two-atom correlation measurement. The second atom is sent through the system following the detection of the first. The passage of this second atom through the apparatus and its detection can be described using the same formal tools adopted for the first atom. Let  $\rho_e^F$  be the trace-normalized reduced densities which describe the state of the field in  $C$  after the detection of the first atom in  $e$  or  $g$ . The conditional probabilities for the various possible results obtained in the final measurement of the state of the second atom, for given outcome of the measurement of the final state of the first atom, are, respectively, given by

$$\begin{aligned} P_{ee} &= \text{Tr}[\mathcal{U}_e^\dagger \mathcal{U}_e \rho_e^F], & P_{ge} &= \text{Tr}[\mathcal{U}_e^\dagger \mathcal{U}_e \rho_g^F], \\ P_{eg} &= \text{Tr}[\mathcal{U}_g^\dagger \mathcal{U}_g \rho_e^F], & P_{gg} &= \text{Tr}[\mathcal{U}_g^\dagger \mathcal{U}_g \rho_g^F]. \end{aligned} \quad (4)$$

Note that in, e.g.,  $P_{ge}$  the first atom is detected in state  $g$  and the second in state  $e$ . Because of the assumed normalization

one has  $P_{ee} + P_{eg} = 1 = P_{ge} + P_{gg}$ , and the correlation signal measured in Ref. [6] can be written simply as

$$\eta = P_{ee} - P_{ge} = \text{Tr}[\mathcal{U}_e^\dagger \mathcal{U}_e (\rho_e^F - \rho_g^F)]. \quad (5)$$

The reduced densities appearing here refer to the state of the field in  $C$  at the time of the passage of the second atom through the apparatus. On the one hand, these expressions connect the two-atom correlation data directly with theoretical models for the fully causal time evolution of  $\rho_g^F$  during the time lapse between the passage of the two atoms. This involves in fact the only truly ‘complex’ aspect of the dynamics, which is responsible for the dissipative evolution of the field state in  $C$ . Since moreover one can always express the reduced densities in terms of their (time dependent) eigenvectors and eigenvalues, Eqs. (4) and (5) also show directly that measured values of the various conditional probabilities and of  $\eta$  can be expressed in terms of these same ingredients. As shown below, the eigenvalues are in fact *determined* by  $P_{ee}$  and  $P_{ge}$ , *provided* specific but ordinary assumptions concerning the dissipation mechanism are made. In this sense, they provide a basis independent characterization of the decoherence process much as quantities based on more general constructs which are nonlinear in  $\rho$  such as, e.g., the ‘idempotency defect’  $1 - \text{Tr} \rho^2$  [9] or entropylike functionals of the density [7]. The assumptions concerning the damping mechanism will allow, in particular, for an *exact* treatment, independently of the also customary recourse to master equation approximations.

In order to model the dissipative effects in  $C$  during the time between the passage of the two atoms, we introduce as usual a set of harmonic oscillators with frequencies  $\omega_k$  and creation and annihilation operators written as  $b_k^\dagger$  and  $b_k$  (the ‘heat bath’) coupled to the relevant mode of the cavity through the bilinear expression

$$H_{\text{int}} = \sum_k \hbar \gamma_k (a^\dagger b_k + b_k^\dagger a). \quad (6)$$

This differs from the also often used position-position coupling by the omission of ‘antiresonant’ terms involving products of two creation and of two annihilation operators. These terms are actually expected to contribute very little to the damping process in view of the smallness of the ratio  $\gamma_k/\omega$  in the present context. The resulting system of coupled oscillators has the well known properties that (i) its ground state is the product of the ground states of the different oscillators (no ground state correlations) and (ii) an initial state written as a product of coherent states, i.e.,

$$|t=0\rangle = |\alpha\rangle \prod_k |\beta_k\rangle \quad (7)$$

retains this form at all times, the amplitudes  $\alpha(t)$  and  $\beta_k(t)$  being solutions to the linear coupled equations

$$i\dot{\alpha} = \sum_k \gamma_k \beta_k, \quad (8)$$

$$i\dot{\beta}_k = (\omega_k - \omega)\beta_k + \gamma_k \alpha,$$

with the given initial conditions, in an interaction picture where the time dependence associated with the constant of motion  $\hbar\omega(a^\dagger a + \sum_k b_k^\dagger b_k)$  has been eliminated. This solution is in fact the relevant one when we then take as initial states the states generated by the action of the operators obtained in Eq. (3) on a coherent state  $|\alpha\rangle$  and a heat bath at temperature zero. Since the state of the relevant mode of  $C$  is then the sum of two linearly independent (though, strictly speaking, never orthogonal) coherent states, we see that we will have at all times a sum of two terms, each being of the type shown in Eq. (7). Consequently, the reduced density describing the state of the field in  $C$  will have its trace exhausted in a (time dependent) two-dimensional subspace of the corresponding oscillator space [10]. This particular feature is in fact retained in the currently used master equation description of the damping process (see, e.g., [5]), which, however, gives quantitatively different results both at very short times (where it gives a linear rather than a quadratic time dependence for decoherence measures such as the idempotency defect [9]) and for long times in the asymptotically damped region.

The computationally simplest case to consider corresponds to case (a) as defined above. We select furthermore the value  $\phi = \pi$  of the phase characterizing the superposition state not only for convenience, but because of the singular properties of this choice, to be discussed below. The “initial” state (after the detection of the first atom) is then

$$|t=0\rangle = N_\alpha [|\alpha\rangle \mp |-\alpha\rangle] \otimes |0_b\rangle,$$

where  $|0_b\rangle$  stands for the “bath” ground state (zero temperature), the double sign corresponds to the detection of the first atom in  $e$  or  $g$ , and  $N_\alpha$  is the appropriate normalization factor. The solution of Eqs. (8) then gives at time  $t$

$$|t\rangle = N_\alpha \left[ |\alpha(t)\rangle \prod_k |\beta_k(t)\rangle \mp |-\alpha(t)\rangle \prod_k |-\beta_k(t)\rangle \right].$$

It is then a simple matter to obtain the  $C$ -field reduced density at time  $t$  as

$$\rho_e^F(t) = N_\alpha^2 \{ |\alpha(t)\rangle \langle \alpha(t)| + |-\alpha(t)\rangle \langle -\alpha(t)| \\ \mp \Gamma_b(t) [|\alpha(t)\rangle \langle -\alpha(t)| + |-\alpha(t)\rangle \langle \alpha(t)|] \}, \quad (9)$$

where the real function  $\Gamma_b(t)$ , which contains the decoherence effects of the bath oscillators, is given by

$$\Gamma_b(t) = \prod_k \langle -\beta_k(t) | \beta_k(t) \rangle.$$

The nonvanishing eigenvalues  $\lambda_\pm(t)$  of each of these two matrices are

$$\lambda_+^{(e)}(t) = \frac{[1 + \Gamma_a(t)][1 \mp \Gamma_b(t)]}{2[1 \pm \Gamma_a(0)]}, \quad (10)$$

$$\lambda_-^{(e)}(t) = \frac{[1 - \Gamma_a(t)][1 \pm \Gamma_b(t)]}{2[1 \pm \Gamma_a(0)]},$$

where  $\Gamma_a(t) = \langle -\alpha(t) | \alpha(t) \rangle$ , with associated eigenvectors

$$|\lambda_\pm(t)\rangle \propto |\alpha(t)\rangle \pm |-\alpha(t)\rangle.$$

The singular character of this special case derives from the fact that the operator  $\exp(i\phi a^\dagger a) = \exp(i\pi a^\dagger a)$  appearing in  $\mathcal{U}_e^\dagger \mathcal{U}_e$  is simultaneously diagonal [with eigenvalues  $\pm 1$  for  $|\lambda_\pm(t)\rangle$ ] with the reduced densities. The correlation signal  $\eta(t)$  measured in [6] then assumes the simple form

$$\eta(t) = \lambda_-^{(e)}(t) - \lambda_+^{(g)}(t).$$

To the extent that the overlap  $\Gamma_a(0)$  is and  $\Gamma_a(t)$  remains negligible, it is clear from Eqs. (10) that  $\lambda_-^{(e)} \approx \lambda_+^{(g)}$ , so that the above result reduces to

$$\eta(t) \approx \lambda_+^{(g)}(t) - \lambda_-^{(g)}(t) = 1 - 2\lambda_-^{(g)}(t) \\ \approx \lambda_-^{(e)}(t) - \lambda_+^{(e)}(t) = 1 - 2\lambda_+^{(e)}(t), \quad (11)$$

which, together with the trace normalization, is sufficient to determine the eigenvalues in this small overlap limit. One sees moreover that, independently of the small overlap condition, the eigenvalues of the reduced densities are determined by the separately measured quantities  $P_{ee}$  and  $P_{ge}$  as  $\lambda_-^{(e)} = P_{ee}$  and  $\lambda_-^{(g)} = P_{ge}$ .

Exactly the same analysis applies in this case when a master equation approximation is used to describe the damping effects associated with Eqs. (8), as done, e.g., in Ref. [5]. In fact, the master equation can be solved in closed form giving for the reduced density an expression identical in form to Eq. (9) with

$$\alpha(t) \rightarrow \alpha(0) e^{-\gamma t/2}$$

and

$$\Gamma_b(t) \rightarrow e^{-2|\alpha(0)|^2(1-e^{-\gamma t})},$$

where  $\gamma = 1/t_c$ , the inverse of the damping time of the field intensity in  $C$ .

Case (b), which corresponds to the conditions of the experiment reported in [6], can be handled in precisely the same way as case (a), since the state of the field prepared in  $C$  (i.e., immediately after the detection of the first atom) is still of the form of a superposition of *just two* coherent states, namely,

$$|t=0\rangle = N_{\alpha\phi} [e^{i\phi} |\alpha e^{i\phi}\rangle + |\alpha e^{-i\phi}\rangle] \otimes |0_b\rangle.$$

Expressions for the two nonvanishing eigenvalues and for the corresponding eigenvectors of the reduced densities can still be calculated in a straightforward way, in terms of the appropriate solutions of Eqs. (8), taking into account the nonorthogonality of the coherent state representation through the appropriate overlap matrix. Since they are considerably

more cumbersome than in the previous case and will not be needed explicitly in the following development we do not give them in full here.

Before turning to a discussion of the general case, we consider the simpler expressions appearing in the small overlap limit (i.e.,  $|\langle \alpha(t)e^{i\phi} | \alpha(t)e^{-i\phi} \rangle| \ll 1$  so that the overlap matrix essentially reduces to the unit matrix). Although the operators involved in  $\mathcal{U}_e^\dagger \mathcal{U}_e$  are no longer simultaneously diagonal with the reduced densities, one still gets for the correlation signal a result similar to Eq. (11), which can be written as

$$\eta(t) \approx \cos \left( \sum_k |\beta_k(t)|^2 \sin 2\phi \right) (\lambda_+ - \lambda_-),$$

where  $\lambda_\pm \approx \lambda_\pm^{(s)} \approx \lambda_\pm^{(e)}$  stand for the small overlap approximations to the eigenvalues of the reduced densities

$$\lambda_\pm = \frac{1 \pm |\Gamma_b(t, \phi)|}{2},$$

with

$$\Gamma_b(t, \phi) = e^{-2\sum_k |\beta_k(t)|^2 \sin^2 \phi} e^{i\theta(t)}$$

and

$$\theta(t) = \sum_k |\beta_k(t)|^2 \sin 2\phi.$$

In the last equations  $\alpha(t)$  and  $\beta_k(t)$  stand for the solutions to Eqs. (8) with initial conditions  $\alpha(0), \beta_k(0) = 0$ . This makes all the dependence on  $\phi$  explicit. The possibility to express the small overlap approximation to  $\eta(t)$  in terms of reduced density eigenvalues is due to the fact that in this limit the two densities  $\rho_e^F$  and  $\rho_g^F$  essentially commute and also have essentially the same eigenvalues, i.e.,

$$\rho_e^F \approx |\lambda_+\rangle \lambda_+ \langle \lambda_+| + |\lambda_-\rangle \lambda_- \langle \lambda_-|,$$

$$\rho_g^F \approx |\lambda_+\rangle \lambda_- \langle \lambda_+| + |\lambda_-\rangle \lambda_+ \langle \lambda_-|,$$

with approximate eigenstates

$$|\lambda_\pm\rangle \propto \pm e^{i(\theta(t)+\phi)} |\alpha(t)e^{i\phi}\rangle + |\alpha(t)e^{-i\phi}\rangle.$$

The correlation signal measured in [6], Eq. (5), thus essentially determines the eigenvalues of the reduced densities  $\rho_e^F$  and  $\rho_g^F$  in the *small overlap limit*. Essential for this result is the pairwise equality of the eigenvalues of the two matrices in this limit. We then finally consider the general case and show that, independently of the small overlap assumption, separate measurements of  $P_{ee}$  and  $P_{ge}$  again determine the eigenvalues of the trace-normalized matrices  $\rho_e^F$  and  $\rho_g^F$ . In fact, it is easy to show that these quantities provide in general independent linear combinations of the two nonvanishing eigenvalues in each case, with coefficients which de-

pend on the dynamics of the atom-field interaction as expressed by matrix elements of the reduced operators  $\mathcal{U}_e$ , Eq. (2), in the reduced density eigenvectors. In order to see this explicitly, it suffices to write the reduced densities in diagonal form as

$$\rho_g^F = |\lambda_+^{(g)}(t)\rangle \lambda_+^{(g)}(t) \langle \lambda_+^{(g)}(t)| + |\lambda_-^{(g)}(t)\rangle \lambda_-^{(g)}(t) \langle \lambda_-^{(g)}(t)|, \quad (12)$$

which gives for the correlation functions

$$P_{ee} = \langle \lambda_+^{(g)}(t) | \mathcal{U}_e^\dagger \mathcal{U}_e | \lambda_+^{(g)}(t) \rangle \lambda_+^{(g)}(t) \\ + \langle \lambda_-^{(g)}(t) | \mathcal{U}_e^\dagger \mathcal{U}_e | \lambda_-^{(g)}(t) \rangle \lambda_-^{(g)}(t).$$

These general expressions hold of course also in case (a) as discussed above, where for  $\phi = \pi$  one has

$$\langle \lambda_+^{(g)} | \mathcal{U}_e^\dagger \mathcal{U}_e | \lambda_+^{(g)} \rangle = 0 \quad \text{and} \quad \langle \lambda_-^{(g)} | \mathcal{U}_e^\dagger \mathcal{U}_e | \lambda_-^{(g)} \rangle = 1.$$

As a result of the fact that the reduced densities are then simultaneously diagonal with the reduced operators  $\mathcal{U}_e$ , the determination of the eigenvalues becomes in fact independent of the structure of the (in this case, common) eigenvectors.

We conclude from these results that the validity of the purely dispersive coupling assumption of the two-level atom to the field in cavity  $C$ , which leads to the validity of Eqs. (1), and the adequacy of a coupling of the form of Eq. (6) to implement the dissipative dynamics allows for a direct experimental monitoring of the intrinsic decoherence processes undergone by the field as a result of its coupling with the ‘‘heat bath.’’ This can be done simply through the separate measurement of the conditional probabilities  $P_{ee}$  and  $P_{ge}$ , since they fully determine the nonvanishing *eigenvalues* (in this case only two, for all times after the preparation of the superposition state) of each one of the reduced densities  $\rho_e^F$  and  $\rho_g^F$ . Under less stringent conditions, allowing for measurable effects of higher-order corrections (in  $\Omega/\delta$ ) to Eqs. (1), one will generate field states, after the passage of the first atom, which cannot be reduced to superpositions of just two linearly independent states (see, e.g., Ref. [4]), and will consequently have reduced densities having a correspondingly higher number of nonvanishing eigenvalues. The representation corresponding to Eq. (12) will then have more than just two terms so that the *complete* determination of intrinsic decoherence properties of the field will require fancier measurements.

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