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Semiclassical dynamics from Zeno-like measurements

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ABSTRACT

The usual semiclassical approximation for atom–field dynamics consists in substituting the field operators by complex numbers related to the (supposedly large enough) intensity of the field. We show that a semiclassical evolution for coupled systems can always be obtained by frequent Zeno-like measurements on the state of one subsystems, independently of the field intensity in the example given. We study the Jaynes–Cummings model from this perspective.

The Quantum Zeno Effect was introduced in the literature by B. Misra and E.C. Sudarshan as a paradox, a theoretical prediction, the freezing of a quantum system by constant measurements, in contradiction with experimental observations. The example presented by the authors is the one of the traces of decaying particles in bubble chambers [1]. In a later publication the same authors solved the so-called "Quantum Zeno Paradox" [2], arguing that the observed tracks were not sufficiently frequent (could not be considered as a continuous measurement), so it could not modify the particle's lifetime.

In 1990, following Cook's proposal [3], the first experimental observation of the Quantum Zeno Effect was realized [4]. The QZE became the center of interesting debates on foundations of Quantum Mechanics [5–7]. This pioneer experiment shows the inhibition of a quantum transition, differently from the initial proposal of B. Misra and E.C. Sudarshan which involved the freezing of a decaying dynamics. Recently an experimental observation of the QZE on decaying systems was reported [8,9].

The QZE is also studied in the context of quantum state protection [10–12], where an increasing number of protocols and strategies proposed have shown the possibilities of practical applications of QZE in quantum information processing.

Frequent measurements do not necessarily freeze the evolution of a quantum system. They can induce an evolution restricted in subspaces defined by the measurements. This evolution has been recently investigated, in Ref. [13], and called quantum Zeno Dynamics. Strategies, based on quantum Zeno dynamics, to protect quantum states from decoherence [14], for state purification [15] and distillation processes [16] have been presented.

In the present contribution we show that the quantum Zeno dynamics in a two degrees of freedom systems will inhibit entanglement and render the dynamics in this sense semiclassical. We study the dynamics of a two level atom interacting with an electromagnetic mode, which is frozen in its initial state by frequent projective measurements. The atomic subspace evolution becomes unitary in the limit of infinite measurements, and depending on the measured field state, the dynamics becomes identical to the semiclassical model for this interaction. Therefore the quantum Zeno dynamics can justify the semiclassical evolution.

Let us start by calculating the time evolution operator for the quantum Zeno dynamics of a bipartite system, composed of subsystems A and B, where subsystem B is frequently measured in its initial state. The Hamiltonian of the system is

$$H = H_A + H_B + H_{AB}, \tag{1}$$

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where H_{AB} is the interaction term. The initial global state is given by:

$$\rho(0) = \rho_A(0) \otimes |B\rangle\langle B|, \tag{2}$$

and its free time evolution (without measurements) can be written as

$$\rho(t) = e^{-\frac{i}{\hbar}[H,\bullet]t}\rho(0), \tag{3}$$

we use the conventional notation for superoperators the dot sign ($\bullet$) indicates the place to be occupied by the state operator where the superoperator acts.

The total dynamics is composed of free evolutions, through short time intervals, followed by projective measurements in the state of subsystem B. The total time of the evolution is $T = tN$ where N is the number of measurements and t is the free evolution time. Notice that the time between measurements is inversely proportional to the number of measurements. We consider a large number of measurements, therefore the free evolution takes place in a very short time interval, and write the series expansion in time for the unitary time evolution up to second order terms $O(t^2)$.

$$\rho(t) = \left(I - \frac{it}{\hbar}[H,\bullet] - \frac{t^2}{2\hbar^2}\big[H,[H,\bullet]\big]\right)\rho(0). \tag{4}$$

The projective measurements in subsystem B are represented by the operator $P_B = I_A \otimes |B\rangle\langle B|$. After one projective measurement $\rho(t)$ can be written as:

$$P_B\rho(t)P_B = \left\{I - \frac{it}{\hbar}\big[\langle H\rangle_B,\bullet\big] - \frac{t^2}{2\hbar^2}\left(\langle H^2\rangle_B\bullet - \langle H\rangle_B\bullet\langle H\rangle_B - \bullet\langle H^2\rangle_B\right)\right\}\rho(0), \tag{5}$$

where $\langle H\rangle_B = \langle B|H|B\rangle$. The expression in (5) can also be written as:

$$\exp\left(-\frac{it}{\hbar}\big[\langle H\rangle_B,\bullet\big] - \frac{t^2}{2\hbar^2}\left(\langle H^2\rangle_B\bullet - \langle H\rangle_B\bullet\langle H\rangle_B - \bullet\langle H^2\rangle_B\right)\right) \times \rho(0) = \left(e^{L_At}\rho_A\right)\otimes|B\rangle\langle B|, \tag{6}$$

notice that $L_A = -\frac{it}{\hbar}[\langle H\rangle_B,\bullet] - \frac{t^2}{2\hbar^2}(\langle H^2\rangle_B\bullet - \langle H\rangle_B\bullet\langle H\rangle_B - \bullet\langle H^2\rangle_B)$ is a superoperator that acts only in subsystem A.

The state operator at time T can be calculated after N applications of e^{L_At} in $\rho(0)$:

$$\rho(T) = \left(e^{L_At}\right)^N\rho(0) = \left(e^{NL_At}\rho_A\right)\otimes|B\rangle\langle B|, \tag{7}$$

as $t = T/N$, we can write

$$e^{NL_At} = \exp\left(-\frac{iT}{\hbar}\big[\langle H\rangle_B,\bullet\big] - \frac{T^2}{N\hbar^2}\left(\langle H^2\rangle_B\bullet - \langle H\rangle_B\bullet\langle H\rangle_B - \bullet\langle H^2\rangle_B\right)\right) \tag{8}$$

and in the limit $N \to \infty$

$$e^{NL_At} = e^{L_AT} = \exp\left(-\frac{i}{\hbar}\big[\langle H\rangle_B,\bullet\big]T\right). \tag{9}$$

The operator e^{NL_At} governs the quantum Zeno dynamics on the bipartite system. Notice that if subsystem B is frozen, the evolution of subsystem A is unitary and depends on the projective measurements in B. The coefficient of second order terms $O(t^2)$ tend to zero as $N \to \infty$, therefore, in this limit there is no entanglement between the systems. Nevertheless, the Hamiltonian interaction term H_{AB} may be significant for the evolution of subsystem A. The expression (9) confirm the results presented in Refs. [15,16], and also the scheme proposed in Ref. [14].

The measurements on subsystem B are relevant for subsystem A only when there is interaction between them ($H_{AB} \neq 0$). Next we show that, for specific parameters, the quantum Zeno dynamics may describe the semiclassical interaction between a two level atom and an electromagnetic mode.

The well-known Jaynes–Cummings model describes the atom–field interaction in the quantum mechanical context. Subsystems that represent the degrees of freedom of the atoms and the field have the dynamics described by a Hamiltonian which allows for excitation transfer between the subsystems. In the semiclassical model for this interaction only the atomic degrees of freedom are described as quantum elements, the electromagnetic field is considered as an external potential responsible for the atomic levels coupling.

We show how to obtain an evolution equivalent to the semiclassical atom–field dynamics from the quantum Zeno dynamics on Jaynes–Cummings model. Following the terminology presented before, let us consider subsystem A describing the atom and subsystem B the field. We have already shown that quantum Zeno dynamics prevent entanglement between the subsystems, and restricts the evolution on subsystem A. Both factors contributed to the construction of the semiclassical interaction from quantum Zeno dynamics.

It is well known that the Jaynes–Cummings model is equivalent to the semiclassical model when the electromagnetic field is prepared in a coherent state with a huge number of photons. The experimental observation of this equivalence was reported in Ref. [17]. This experiment consists in a atomic interferometer, where a field mode, prepared in a coherent state and interacting with the atoms, act as a "beam-splitter". In the quantum regime (where the mean photons number is low $|\alpha|^2 \approx 1$) the field is able to register the atomic "path", the atom–field interaction produces an entangled state. In classical regime (where the mean photon number is high) the microscopic "beam-splitter" does not record the particle's path. The field state remains basically the same before and after the interaction with the two level atom, the atomic energy is not sufficient to modify the field state, this is an essential element that allows for a semiclassical dynamics between atom and field.

In the quantum Zeno dynamics of the Jaynes–Cummings model the field state also remains in its initial state. Frequent projective measurements inhibits the field evolution, therefore, as in the semiclassical dynamics, the atom interacts with a system that does not evolve, and can be seen as an external perturbation in the atomic evolution.

The Jaynes–Cummings Hamiltonian, in RWA, is given by

$$H_{JC} = \frac{1}{2}\hbar\omega_a\sigma_z + \hbar\omega a^\dagger a + g\big[a\sigma_+ + a^\dagger\sigma_-\big], \tag{10}$$

where ω_a is the atomic transition frequency, ω is the electromagnetic field frequency and g is the coupling coefficient.

As shown in Eq. (9) the atomic subspace evolution depends on the state $|B\rangle$ where the field is measured. If the field is projected in the state $|B\rangle = |\alpha\rangle$, the Hamiltonian $\langle H_{JC}\rangle_B$ that governs the dynamics in (9) can be written as

$$\langle H_{JC}\rangle_B = \frac{1}{2}\hbar\omega_a\sigma_z + g\big[\alpha\sigma_+ + \alpha^*\sigma_-\big], \tag{11}$$

that is the exact same form of the Hamiltonian in semiclassical description of mater and light interaction. Notice that this "classical limit" is valid for any coherent state mean photon number, as long as the field state has its evolution inhibited by projective measurements, the atomic dynamics is given by the semiclassical Hamiltonian (11). The time evolution operator of the quantum Zeno evolution, deduced in Eq. (9), governed by Hamiltonian (11) justify the atom–field semiclassical dynamics.

If the field is projected in any linear combination of consecutive Fock states, as $|B\rangle = \cos(\theta)|n\rangle + e^{i\phi}\sin(\theta)|n+1\rangle$, the Hamiltonian responsible for the time evolution on the atomic subsystem is given by

$$\langle H\rangle_B = \frac{1}{2}\hbar\omega_a\sigma_z + g\big[\cos(\theta)e^{i\phi}\sin(\theta)\sqrt{n+1}\,\sigma_+ + \cos(\theta)e^{-i\phi}\sin(\theta)\sqrt{n+1}\,\sigma_-\big]. \tag{12}$$

The evolution governed by (12) is similar to the one in (11).

On the other hand, if the field is measured in a Fock state, as $|B\rangle = |n\rangle$, the Hamiltonian $\langle H_{JC}\rangle_B$, does not couple the atomic states any more.

$$\langle H_{JC}\rangle_B = \langle n|\left(\frac{1}{2}\hbar\omega_a\sigma_z + \hbar\omega a^\dagger a + g\big[a\sigma_+ + a^\dagger\sigma_-\big]\right)|n\rangle = \frac{1}{2}\hbar\omega_a\sigma_z + \hbar\omega. \tag{13}$$

The reason is that the coupling term in H_{JC} is linear, therefore the average in Fock states on it is null.

The divergence between the evolution generated by projections of the field in the states $|n\rangle$, $|\alpha\rangle$ and $\cos(\theta)|n\rangle + e^{i\phi}\sin(\theta)|n+1\rangle$ are related to the information obtained on subsystem B. Projections in $|n\rangle$ represents complete information about the number of photons in the electromagnetic mode. Therefore, the dynamics composed by frequent measurements ($N \to \infty$) on subsystem B in $|n\rangle$ conserves this subsystem in a defined photon number state. This dynamics does not allow for excitation exchange between atoms and the field mode. On the other hand, projections in $|\alpha\rangle$ or $\cos(\theta)|n\rangle + e^{i\phi}\sin(\theta)|n+1\rangle$ represents incomplete information about the photons number, and excitations exchange are possible.

To summarize, we have shown that the quantum Zeno dynamics can justify a semiclassical Hamiltonian for the atom–field interaction even in the small photon number limit. Projective measurements in the field state induce an unitary evolution on atomic subspace. For appropriate choices of the measured states this unitary evolution is identical to the one governed by a semiclassical Hamiltonian.

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