Correlation-based method for comparing and reconstructing nearly identical two-dimensional structures

Yobani Mejía-Barbosa

A method for comparing and reconstructing two nearly identical planar objects that are composed of the same number of identical apertures is presented. These structures differ only in the location of one of the apertures. The method is based on a subtraction algorithm, which involves the cross-correlation and autocorrelation functions of the compared structures. Simulated results illustrate the feasibility of the method. © 2001 Optical Society of America

OCIS codes: 070.2580, 070.5010, 070.6020.

1. Introduction

Optical correlation techniques are frequently used to detect signals in noise, in pattern recognition, and so forth.1,2 A common task in pattern recognition is the comparison of two or more structures, to search either for defects or for identification. Objects to be compared can be periodic or quasi-periodic structures, for example, photomasks for integrated circuit manufacture, crystalline gratings, and diffraction gratings. The comparison3 and reconstruction4–6 of the structures can be carried out by analysis of their autocorrelation functions, which can be obtained by calculation of the Fourier transform of the intensity distribution of the Fraunhofer diffraction patterns of the structures, i.e., the Wiener–Khintchine theorem.7,8

In this paper a method to identify the differences between two similar structures and to reconstruct them is proposed. This method is based on the difference between the cross-correlation and the autocorrelation functions of the structures. Each structure consists of a two-dimensional arrangement of identical apertures whose transmittance function is a constant and real (amplitude only). The unique difference between the structures to be compared consists in the location of one of the apertures [Figs. 1(a)–1(c)]. The cross-correlation and autocorrelation functions of the structures can be obtained numerically by use of the fast Fourier transform or experimentally with, for example, a VanderLugt filter.9

2. Theory

Let us consider two similar structures to be compared, each with N identical square apertures distributed on an opaque screen as in Figs. 1(a)–1(c). The transmittance of any aperture will be $A \text{rect}(x/a)\text{rect}(y/a) \ast \delta(x - x_n, y - y_n)$, where $0 \leq A \leq 1$ is the aperture transmittance, $a$ is the side length of the aperture, and $x_n$, $y_n$ are the coordinates of the center of the $n$th aperture. The function $\text{rect}(\cdot)$ takes the value 1 inside the aperture and null otherwise, the symbol $(\ast)$ denotes the convolution operator, and $\delta(\cdot)$ is the Dirac delta function. Then the transmittance of a structure will be given by

$$t(x, y) = A \text{rect}(x/a)\text{rect}(y/a) \ast \sum_{n=1}^{N} \delta(x - x_n, y - y_n)$$

$$= t_0(x, y) \ast \sum_{n=1}^{N} \delta(x - x_n, y - y_n).$$  (1)

By illumination of the structure with a collimated optical field of wavelength $\lambda$, the intensity distribution of the Fraunhofer pattern can be obtained at the...
is the Fourier transform of $I$ with the geometrical coordinates $x$, $y$.

According to the Wiener–Khintchine or autocorrelation theorem, the Fourier transform of $I(u, v)$ yields the autocorrelation of the transmittance $t(x, y)$; i.e.,

$$
\Gamma_{tt}(x, y) = \Gamma_0(x, y)
$$

$$
\ast \sum_{m=1}^{N} \sum_{n=1}^{N} \delta[x - (x_m - x_n), y - (y_m - y_n)],
$$

where $\Gamma_0(x, y)$ exhibits a pyramidal shape because it is the Fourier transform of $I_0(u, v)$. In practical situations, the autocorrelation $\Gamma_{tt}(x, y)$ can be obtained experimentally by use of a VanderLugt filter or numerically by calculation of the Fourier transform of the intensity distribution of the Fraunhofer interferogram (Fig. 2) after its capture by a conventional CCD array.

Now let us consider two similar structures of transmittance functions $f(x, y)$ and $g(x, y)$ that differ only in the location of one aperture, for example, the $k$th aperture, so that the coordinates of their apertures satisfy the condition $(x_n^f, y_n^f) = (x_n^g, y_n^g)$ for $n \neq k$. The superscripts correspond to each structure. As a consequence, $x_n^g - x_n^f = x_n^g - x_n^g$ and $y_n^g - y_n^f = y_n^g - y_n^g$ for $n \neq k$. So the cross correlation between such structures (Fig. 3) is given by

$$
\Gamma_{fg}(x, y) = \Gamma_0(x, y)
$$

$$
\ast \sum_{m=1}^{N} \sum_{n=1}^{N} \delta[x - (x_m^g - x_n^f), y - (y_m^g - y_n^f)].
$$

Once we have the autocorrelations of the structures to be compared and their cross correlation, we calculate numerically the subtractions $\Gamma_{gf}(x, y) - \Gamma_{fg}(x, y)$ and $\Gamma_{fg}(x, y) - \Gamma_{gg}(x, y)$. The first one provides information about $f(x, y)$ and the second one about $g(x, y)$, as we will show in the following.

Taking into account the previous condition be-

---

Fig. 1. (a–c) Similar structures of nine elements (identical square apertures of side $a$ and transmission $A$). In comparison with (a), in (b) one aperture was shifted on the right a distance equal to the side $a$, and in (c) one aperture was shifted on the diagonal a distance equal to the aperture diagonal. (d–f) Intensity distributions of the corresponding Fraunhofer interferograms.
between the aperture coordinates, these subtractions yield
\[
\Gamma_{fg}(x, y) - \Gamma_{ff}(x, y) = \Gamma_0(x, y) * \left[ \delta(x - \Delta x_{mk}^f, y - \Delta y_{mk}^f) \right]
\]
respectively. The coordinates of the \( k \)th apertures of both structures are related by \( x_k^g = x_k^f + \varepsilon_k \) and \( y_k^g = y_k^f + \eta_k \), where either \( \varepsilon_k \) or \( \eta_k \) or both are not zero. Thus Eqs. (5a) and (5b) become
\[
\begin{align*}
\Gamma_{fg}(x, y) - \Gamma_{ff}(x, y) &= \Gamma_0(x, y) * \left[ \delta(x - \Delta x_{mk}^f, y - \Delta y_{mk}^f) \right] \\
\Gamma_{gg}(x, y) - \Gamma_{ff}(x, y) &= \Gamma_0(x, y) * \left[ \delta(x - \Delta x_{mk}^g, y - \Delta y_{mk}^g) \right],
\end{align*}
\]
with \( \Delta x_{mk}^f = x_m^f - x_n^f \) and \( \Delta y_{mk}^f = y_m^f - y_n^f \), respectively. The coordinates of the \( h \)th apertures of both structures are related by \( x_h^g = x_h^f + \varepsilon_h \) and \( y_h^g = y_h^f + \eta_h \), with either \( \varepsilon_h \) or \( \eta_h \) or both are not zero. Thus Eqs. (6a) and (6b) become
\[
\begin{align*}
\Gamma_{fg}(x, y) - \Gamma_{ff}(x, y) &= \Gamma_0(x, y) * \left[ \delta(x - \Delta x_{mk}^f, y - \Delta y_{mk}^f) \right] \\
\Gamma_{gg}(x, y) - \Gamma_{ff}(x, y) &= \Gamma_0(x, y) * \left[ \delta(x - \Delta x_{mh}^g, y - \Delta y_{mh}^g) \right],
\end{align*}
\]
with \( \Delta x_{mh}^g = x_m^g - x_n^g \) and \( \Delta y_{mh}^g = y_m^g - y_n^g \).
structure $f(x, y)$, and $\delta(x - \varepsilon_k, y - \eta_k) - \delta(x, y)$ [Figs. 4(a) and 5(a), middle pictures], which provides the information about the difference vector in the locations of the $k$th aperture of the compared structures. The shape of the correlation peaks of the difference $\Gamma_{fg}(x, y) - \Gamma_{gg}(x, y)$ in Eq. (6b) is also given by $\Gamma_0(x, y)$, but now their distribution results from the convolution between $\sum_{m=1}^N \delta(x - \Delta x_{mk}, y - \Delta y_{mk})$ [Figs.

**Fig. 4.** Subtraction algorithm for comparing the structures $f(x, y)$ [Fig. 1(a)] and $g(x, y)$ [Fig. 1(b)]. (a) $\Gamma_{fg}(x, y) - \Gamma_{ff}(x, y)$, and (b) $\Gamma_{fg}(x, y) - \Gamma_{gg}(x, y)$.

**Fig. 5.** Subtraction algorithm for comparing the structures $f(x, y)$ [Fig. 1(a)] and $g(x, y)$ [Fig. 1(c)]. (a) $\Gamma_{fg}(x, y) - \Gamma_{ff}(x, y)$, and (b) $\Gamma_{fg}(x, y) - \Gamma_{gg}(x, y)$. 
4(b) and 5(b), right-hand pictures, which represents the aperture distribution of the structure $g(x, y)$, and 
\[
\delta(x - \varepsilon_k, y - \eta_k) - \delta(x, y) \quad \text{[Figs. 4(b) and 5(a), middle pictures].}
\]

Thus this method enables us not only to identify the difference between two quasi-identical structures to be compared but also to reconstruct them when the unique difference between the structures is the location of one aperture.

3. Conclusions

Similar amplitude-only structures composed of a two-dimensional arrangement of identical apertures can be compared by their autocorrelation functions. If the unique difference between the structures is the location of one aperture, it is possible to reconstruct the structures and identify their differences by calculation of the difference between the cross-correlation function of the structures and their autocorrelation functions. This method can be applied when the correlation functions are obtained numerically or experimentally from the intensity distribution of the diffraction patterns produced by the structures.

This paper was finished at the Abdus Salam International Centre for Theoretical Physics. I thank the encouragement and helpful discussions of Román Castañeda (Universidad Nacional de Colombia Sede Medellín, Medellín, Colombia). I also thank William T. Rodes (Georgia Tech, Lorraine, France) for his excellent comments. This study was performed with support from the Ph.D. Fellow Program of Ciencias (Colombia).

References


10 January 2001 / Vol. 40, No. 2 / APPLIED OPTICS 239